CIVIL-312: Hydraulic Engineering and Infrastructures

Instructor: Prof. Dr. Mirko Musa
Sustainable River Engineering, Energy, & Morphodynamics (STREEM)
GC G1 517 (Bâtiment GC)
E-mail: mirko.musa@epfl.ch

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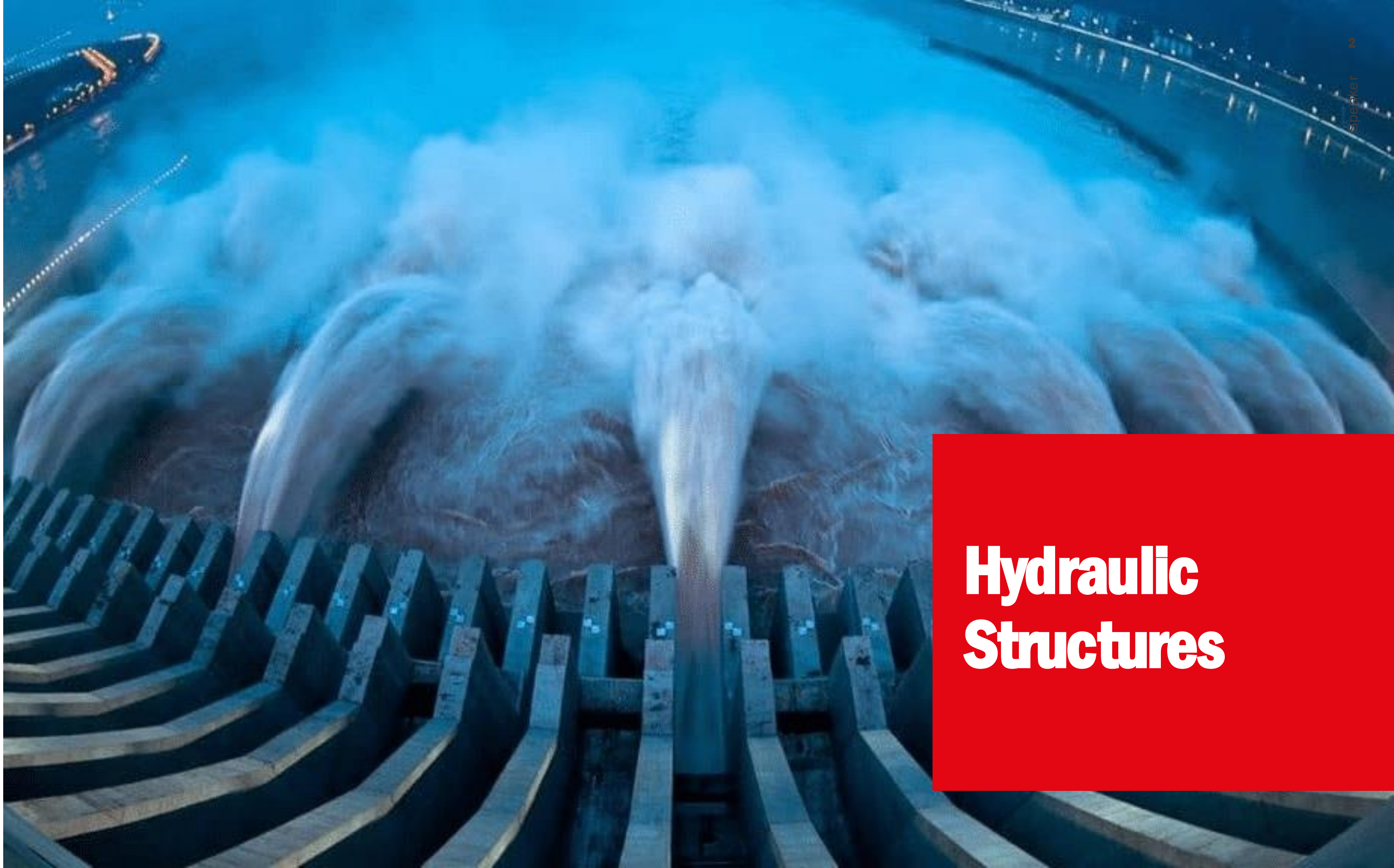


Hydraulic Structures

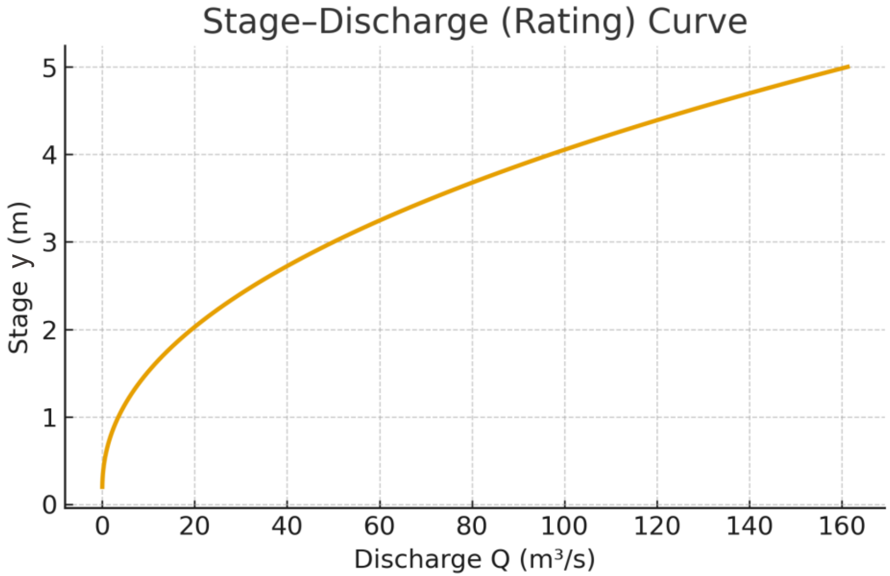
Channel Design & Culverts

Channel Design – an overview

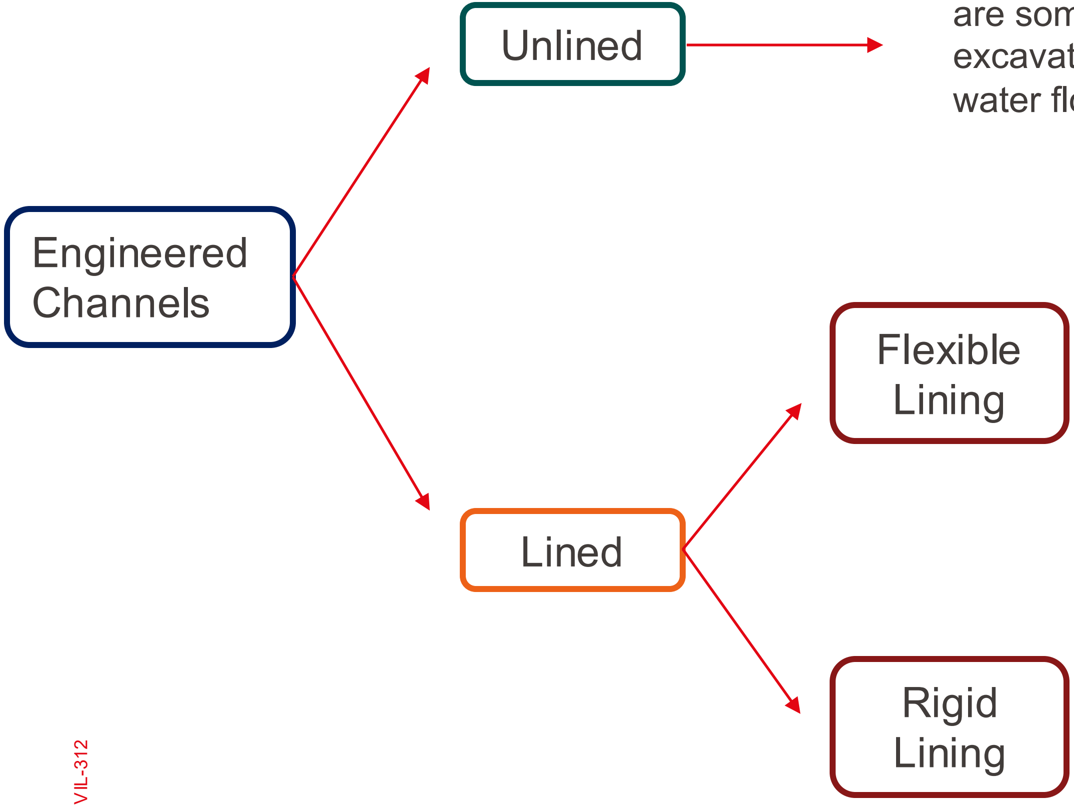
- The main purpose a conveyance structure (canal or gallery) is to bring water from location A to location B
- They are designed to carry a flow discharge Q with a given designed cross-section and slope.
- $Q = f(y)$

The discharge used for the design depends on the purpose of the channel.

For example: drainage channels are usually dry when there is no surface runoff and are designed to accommodate the peak runoff rate from a design storm.



Type of channels



are sometimes called earthen channels, are simply excavated channels in the ground through which water flows

include *riprap*, *gravel*, *vegetation*, manufactured *mats*, or combinations of these materials. They are called “flexible” because they are able to conform to changes in channel shape while maintaining the overall integrity of the channel lining

include *concrete*, *stone masonry*, *soil cement*, *grouted riprap*, and *precast interlocking blocks*. They are are called “rigid” because they tend to crack when deflected

Type of channels

Lining type choice depends on the purpose of the channel.

Lined = NON-ERODIBLE

- Less resistance to flow
- Used to control erosion.
- high velocities (to reduce construction and excavation costs)
- decrease seepage losses
- decrease operation and maintenance costs
- ensure the stability of the channel section.
- Used where high level of safety are required.
- requires specialized equipment and relatively costly material, hence the cost is high.

Example of applications:
- Drainage
- Water supply
- Hydropower

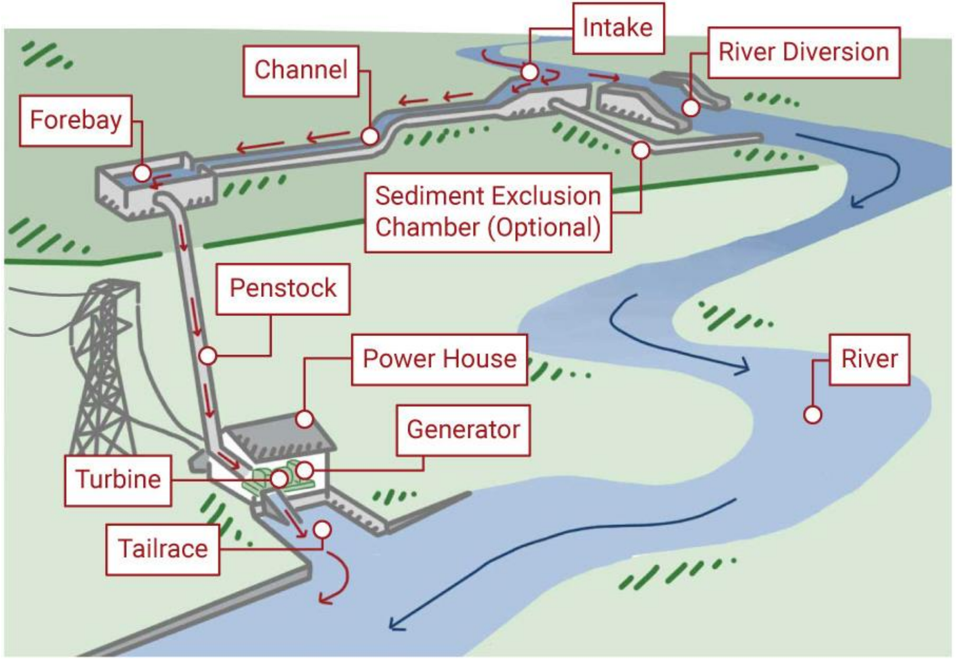
Unlined = ERODIBLE

- less expensive
- permit infiltration and exfiltration
- filter out contaminants
- provide better habitat opportunities
- have a natural appearance.
- However, channels with flexible linings have the disadvantage of being limited in the magnitude of the erosive force that they can sustain without damage to either the channel or the lining.

Example of applications:
- Drainage
- Irrigation

Type of channels

- **Hydropower**
  - Need to reduce hydraulic losses as much as possible
  - High level of operational safety
  - High and constant costs throughout the year
  - Low bottom slopes, low roughness, safe linings



Type of channels

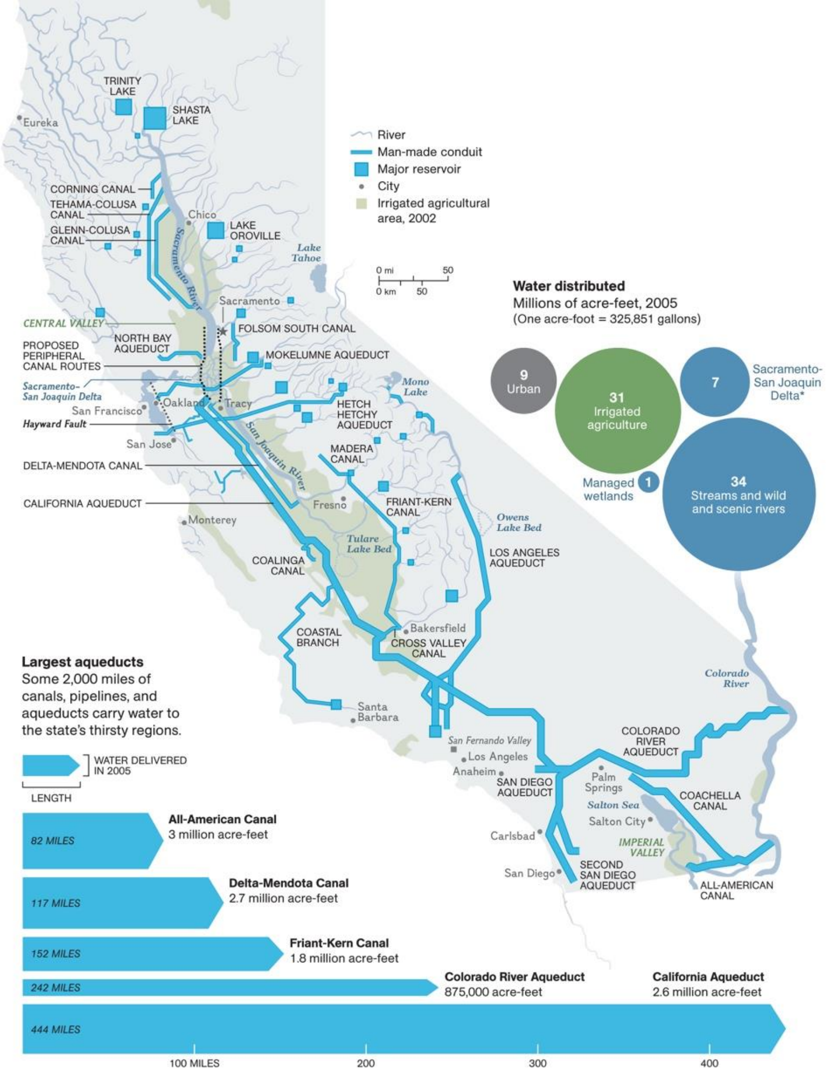
- **Drainage**
  - Urban drainage: channel designed for peak flow rate from a runoff event with a specified return period
  - Rainwater drainage
  - Sewer network
  - Rural drainage

Rip-rap lining

Concrete lining

Type of channels

- **Water Supply**
  - Discharge chosen to meet supply and demand
  - Check structures to maintain the level and ensure correct operations to satisfy demand
  - Several water withdrawal point along the channel



Type of channels

- **Irrigation**
  - Variable demand
  - Both lined and unlined channels

General Criteria

- Must function according to hydraulic criteria
- Be economical (benefits/costs)
- Environmentally acceptable
- Follow specific socio-political criteria (policy)

Channels are designed to operate in **uniform flow conditions**

The channel section/design is selected according to main restrictions:

- Hydraulic
- Construction
- Cost
- That meet the ultimate purpose

Best Hydraulic Section

The best hydraulic section, also known as the most economic section, **is the channel shape that carries the required flow using the *smallest possible flow area***

Why?
→ COST!!

Consider Manning:

$$Q = \frac{1}{n} A\, R_H^{2/3} S_0^{1/2}$$

where the hydraulic radius is defined as $R_H = A/P$ (with P the wet perimeter).
So you can see that:

$$A = \left(\frac{nQ}{S_0^{1/2}}\right)^{3/5} P^{2/5}$$

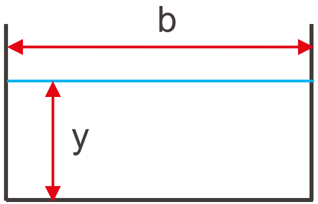
$A \propto P$ so minimizing A means to minimize P.

The best hydraulic section is defined as the section that minimizes the flow area for given values of Q, S_0 and n

Theoretically, **the semicircle is the optimum cross-section in terms of hydraulic capacity BUT it is not practical to construct**.

→ In practice, all cross-sectional shapes that circumscribe a semicircle are said to be "optimal”.

For example, for a rectangular cross-section:



$$A = by$$

$$P = 2y + b = 2y + \frac{A}{y}$$

Minimize the wet perimeter

$$\frac{dP}{dy} = 0 \Rightarrow \frac{-A}{y^2} + 2 = 0$$

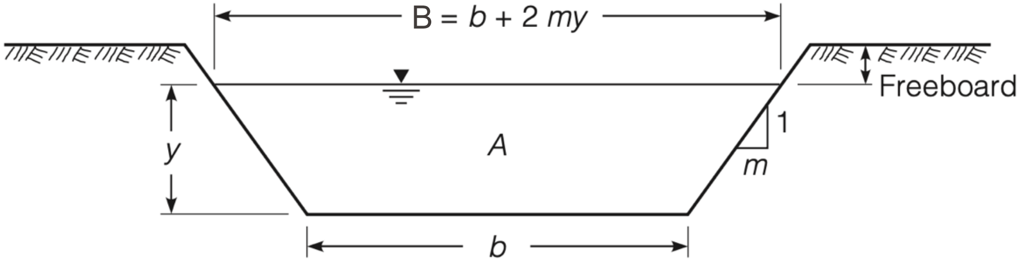
$$\frac{A}{y^2} = 2 \Rightarrow \frac{by}{y^2} = 2$$

$$y = \frac{1}{2} b$$

And of course, y is the estimated uniform depth because channels are designed for uniform flow conditions

A rectangular section is most efficient when the flow depth is one-half the channel width

For a trapezoid, you can show that:



$$A = by + my^2$$
$$P = b + 2y\sqrt{m^2+1}$$

Using b to substitute the first expression into the second it leads to:

$$A = \left(P - 2\sqrt{m^2+1}\right)y + my^2 \longrightarrow$$

and substituting
A using Manning:

$$\underbrace{\left(P - 2y\sqrt{m^2+1} - m\right)y + my^2 = \left(\frac{nQ}{S_0^{1/2}}\right)^{3/5} P^{2/5}}$$

- Holding m constant, taking the partial derivative of each term of that expression in y and setting $\frac{\partial P}{\partial y} = 0$ leads to $\longrightarrow$ $P = 4y\sqrt{m^2+1} - 2my$
- Holding y constant, taking the partial derivative of each term of that expression in m and setting $\frac{\partial P}{\partial m} = 0$ leads to $\longrightarrow$ $m = \frac{1}{\sqrt{3}} \approx 0.577$

And on the basis of those two expressions, the **best hydraulic section for a trapezoid** is one having the following geometric characteristics:

$$\boxed{P = 2\sqrt{3}\,y} \qquad \boxed{b = 2\frac{\sqrt{3}}{3}\,y} \qquad \boxed{A = y^2\sqrt{3}}$$

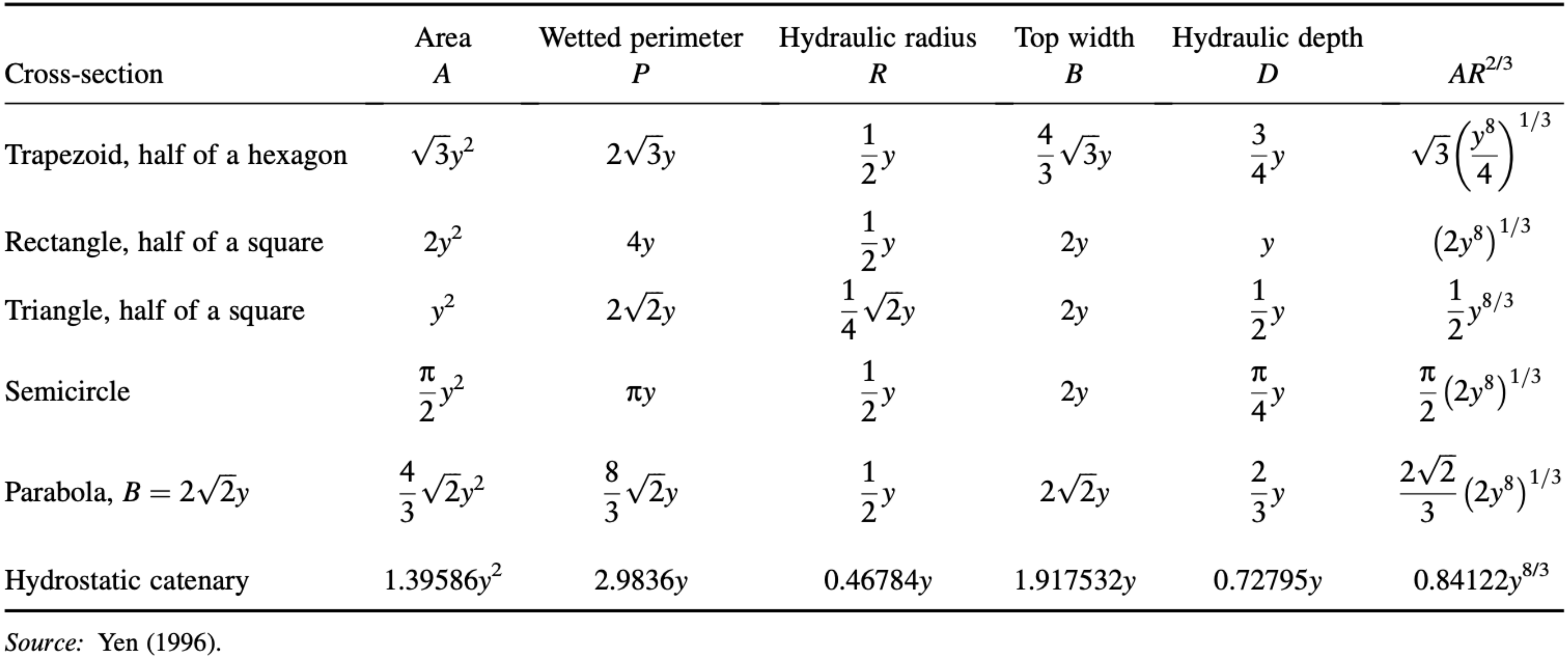
Which shows that $P = 3b$ indicating that the (wetted) sides of the channel have the same length as the bottom. Also, $m = 1/\sqrt{3}$ means that the best side slope is $\theta = 60°$

In cases where the side slopes are controlled by the angle of repose of the soil surrounding the channel (thus it DICTATES m), this $P = 4y\sqrt{m^2+1} - 2my$ combined with $P = b + 2y\sqrt{m^2+1}$ is used to find the best width-to-depth ratio for the selected side slope m:

$$\frac{b}{y} = 2\left(\sqrt{m^2+1} - m\right)$$

Again, channels are designed according to uniform flow conditions, so the reference water depth y would be the uniform depth.

List of Best Hydraulically Efficient Sections Without Freeboard



Cross-section	Area A	Wetted perimeter P	Hydraulic radius R	Top width B	Hydraulic depth D	$AR^{2/3}$
Trapezoid, half of a hexagon	$\sqrt{3}y^2$	$2\sqrt{3}y$	$\frac{1}{2}y$	$\frac{4}{3}\sqrt{3}y$	$\frac{3}{4}y$	$\sqrt{3}\left(\frac{y^8}{4}\right)^{1/3}$
Rectangle, half of a square	$2y^2$	$4y$	$\frac{1}{2}y$	$2y$	y	$(2y^8)^{1/3}$
Triangle, half of a square	y^2	$2\sqrt{2}y$	$\frac{1}{4}\sqrt{2}y$	$2y$	$\frac{1}{2}y$	$\frac{1}{2}y^{8/3}$
Semicircle	$\frac{\pi}{2}y^2$	πy	$\frac{1}{2}y$	$2y$	$\frac{\pi}{4}y$	$\frac{\pi}{2}(2y^8)^{1/3}$
Parabola, $B = 2\sqrt{2}y$	$\frac{4}{3}\sqrt{2}y^2$	$\frac{8}{3}\sqrt{2}y$	$\frac{1}{2}y$	$2\sqrt{2}y$	$\frac{2}{3}y$	$\frac{2\sqrt{2}}{3}(2y^8)^{1/3}$
Hydrostatic catenary	$1.39586y^2$	$2.9836y$	$0.46784y$	$1.917532y$	$0.72795y$	$0.84122y^{8/3}$

Source: Yen (1996).

Velocity and Shear stress Considerations

Another important parameter to consider when designing channels are the min/max velocity expected and allowed and the consequent shear stress exerted at the bottom.

- A **maximum velocity** limit is typically need for unlined channel to avoid erosion (for lined channel typically it’s not necessary because unerodable)
  - If particles are transported, even lined channel can be abraded. Each lining material have a maximum velocity to resist to abrasion (e.g., for concrete, velocity should not exceed 4-10 m/s)

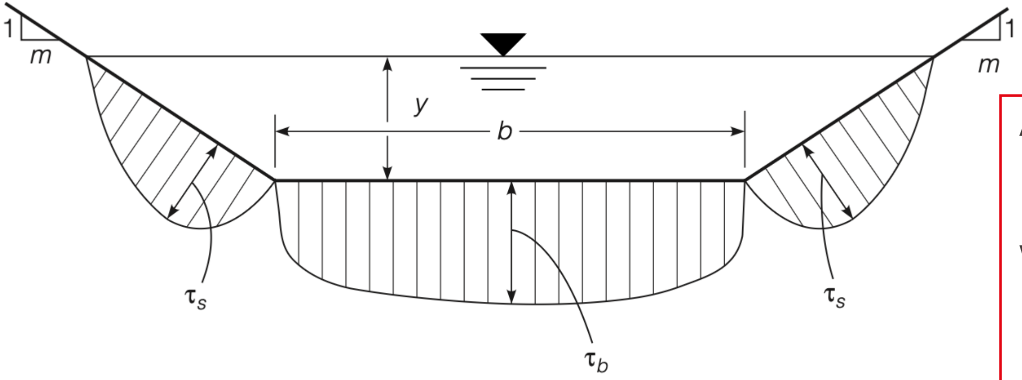
- A **minimum velocity** might be necessary to avoid *siltation* (sediment deposition) and/or to avoid vegetation/algal growth
  - Rule-of-thumb: minimum velocity should not be less than 0.5 - 0.75 m/s

Associated to the flow velocity is the concept of the *shear stress* at the bottom as we saw in previous module. As a reminder, the average shear stress, τ_0 [force/area], on the perimeter of an open channel in uniform flow condition is given by:

$$\tau_0 = \gamma R_H S_0$$

This is also called the *unit tractive force.* Depending on the shape of the cross section, the structure of secondary flow cells (transverse to the main-flow direction), and the nonuniformity in the boundary roughness, this may be non-uniformly distributed along the perimeter.

For a trapezoid for example:



At the bottom:

$$\tau_b = \gamma y S_0$$

At the sides:

$$\tau_s = K_s \tau_b$$

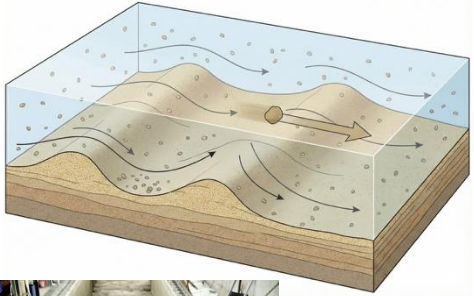
with

$$K_s = \begin{cases} 0.77, & m \leq 1.5 \\ 0.066\,m + 0.67 & 1.5 < m < 5 \\ 1 & m \geq 5 \end{cases}$$

- Estimation of the maximum shear stresses exerted on the bottom and sides of a channel is fundamental to designing open channels, since **channels are stable when the perimeter shear stress is everywhere less than or equal to the shear stress required to move or dislodge the material on the perimeter of the channel**.

- The shear stress required to dislodge the perimeter material is commonly called the *maximum permissible shear stress*, the *critical shear stress*, or simply the *permissible shear stress*. The maximum permissible shear stress is specific to the lining material in the channel.

- For unlined channels, the critical shear stress depends on the property of the soil (sediments). If the shear stress is above critical shear stress, sediments are eroded and ***sediment transport occurs*** (typical in natural rivers).



Channel Slopes

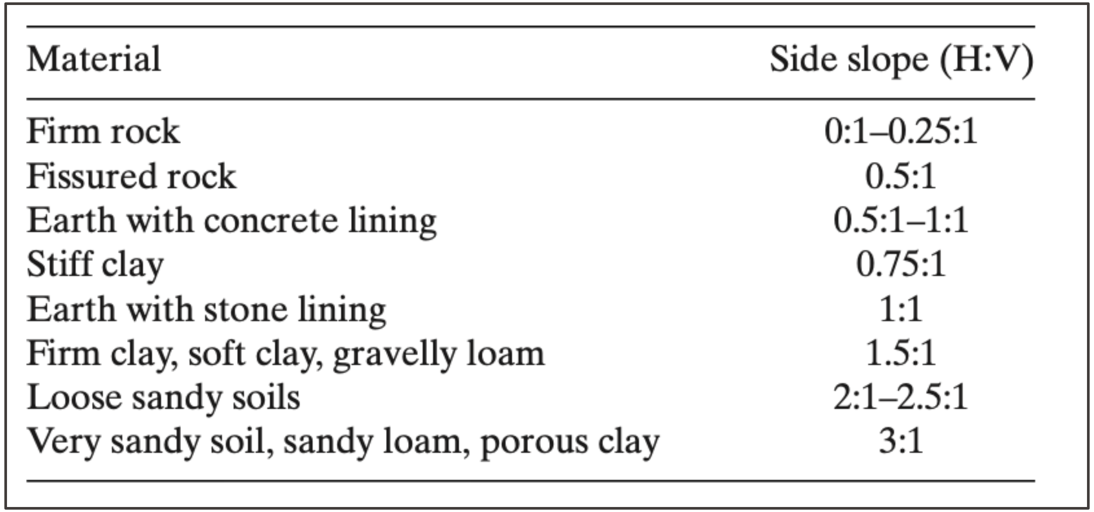
Two key slopes in channel design: longitudinal slope (along the channel) and side slope (cross-section)

- **Longitudinal slope:** usually matches natural ground slope to minimize excavation, but must also prevent shear stress from exceeding lining limits

- **Side slopes:** depend on the soil/material type; steeper slopes often allowed above the waterline in deep cuts

➢ **Concrete-lined channels:** slopes steeper than 1:1 require forms; steeper than 0.75:1 require structural design for earth pressures

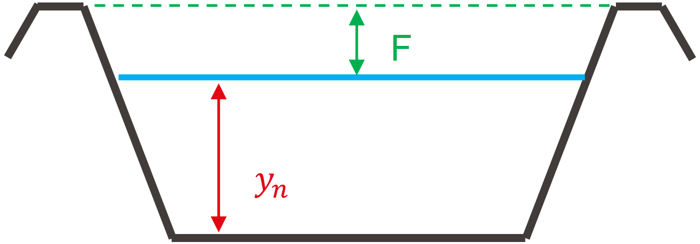
➢ **Typical recommendations:** ~1.5:1 for concrete canals (USBR) and no steeper than 3:1 for roadside channels (FHWA)



Material	Side slope (H:V)
Firm rock	0:1–0.25:1
Fissured rock	0.5:1
Earth with concrete lining	0.5:1–1:1
Stiff clay	0.75:1
Earth with stone lining	1:1
Firm clay, soft clay, gravelly loam	1.5:1
Loose sandy soils	2:1–2.5:1
Very sandy soil, sandy loam, porous clay	3:1

Freeboard

- The **freeboard** is defined as the vertical distance between the water surface and the top of the channel **when the channel is carrying the design flow rate at normal depth**
- Freeboard is provided to account for the uncertainty in the design, construction, and operation of the channel limits
- At a minimum the freeboard should be sufficient to prevent waves or fluctuations in the water surface from overflowing the sides.



An example of recommendation is:

$$F = \begin{cases} 0.15\,m & y < 0.30\,m \\ 0.3\,m & y \geq 0.3\,m, V \leq 1.72\,m/s \\ (0.15 + V^2/2g)m & y \geq 0.3\,m, V > 1.72\,m/s \end{cases}$$

And at **channel bends**, additional freeboard must be provided to accommodate the superelevation of the water surface $h_s = V^2B/(g\,r_c)$, with B the top width and r_c the radius of curvature.

Typical design procedure for Rigid Linings

1. Estimate the roughness coefficient n for the specified lining material. Guidance for estimating Manning's coefficient n for rigid boundary channels are typically tabulated.

2. Compute the normal depth of flow, $y\ [m]$, using the Manning equation selecting the best hydraulic section geometry if appropriate and S_0 based upon topography and other considerations such as alignment. The flow discharge to carry is specified by design.

3. Check the minimum velocity if water carries silt.

4. Estimate the required freeboard and increase the freeboard in channel bends as appropriate to account for superelevation.

Typical design procedure for Flexible Linings

The procedure for flexible-lining materials used in practice, such as rolled erosion control products (RECPs), vegetative lining, riprap, and gabions, is slightly different because the permissible shear stress must be verified in order to prevent dislodging/erosion.

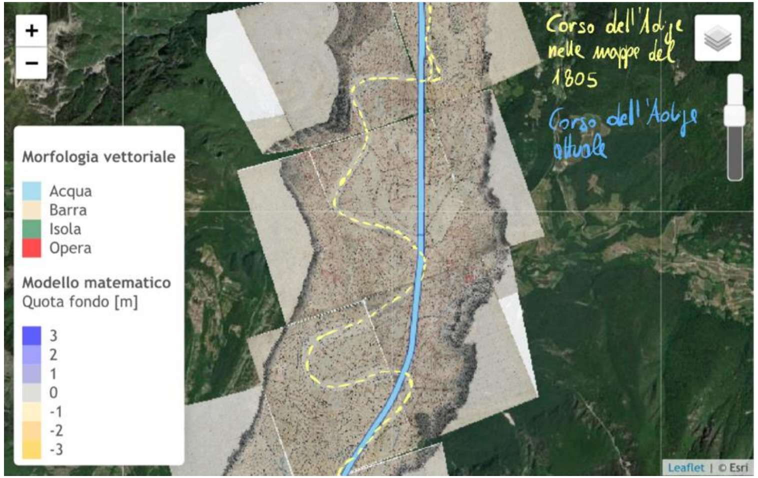
The approaches used to estimate n, τ, and τ_p can vary significantly between linings and flow conditions; however, the design procedure for all flexible linings is similar.

Again, the discharge to carry is defined by design and the slope S_0 is typically based on topography.

1. Select the lining type.
2. Calculate the normal depth $y\ [m]$ considering that the relationship between n and y might be a property of the selected lining.
3. Calculate the maximum shear stress on the perimeter of the channel using the τ relationships seen before. The bottom and side τ might differ.
4. Estimate the permissible shear stress on the perimeter of the channel τ_p.
5. If the design shear stress is ***less than*** τ_p the lining is acceptable. If the lining is inadequate, repeat Steps 1 to 5 for different linings until an adequate lining is found. Several adequate linings might be identified. The preferred lining is selected by considering factors such as cost and practicality.

Stream Restoration?

- Rivers were channelized to gain land for development, control flooding, improve navigability, etc.
- Loss of morphodynamics heterogeneity and biodiversity
- Can we safely invert this trend?



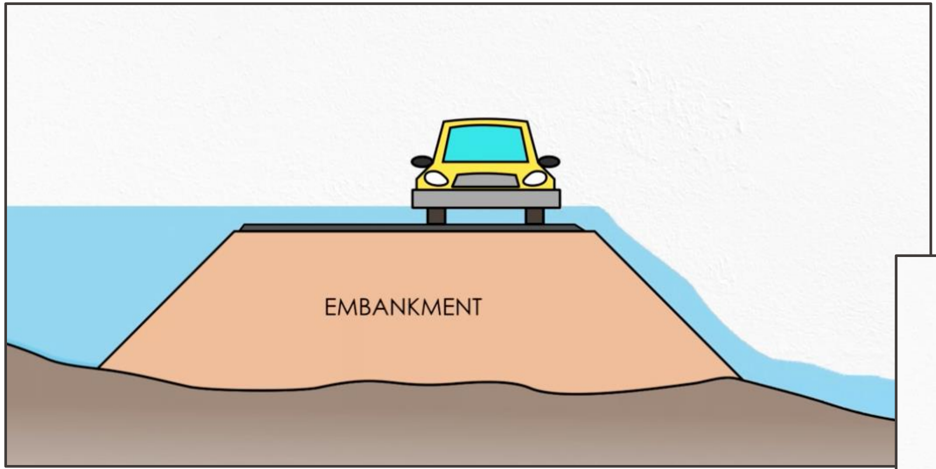
Culverts

Culverts (fr: Ponceaux)

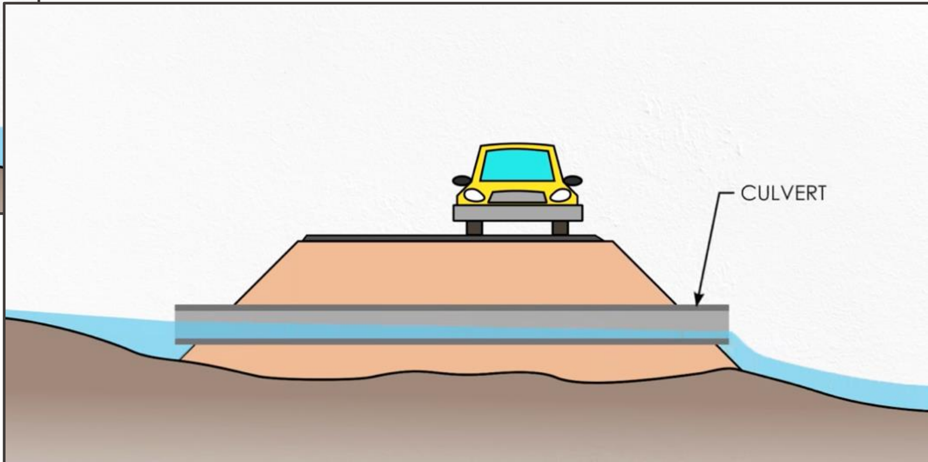
https://www.youtube.com/watch?v=15XJDmawbYU&t=1s

We build a culvert whenever we need to build a road, railway, or embankment across a path where water naturally flows and when a whole bridge would not make sense economically (e.g., go over a small creek).

Without a culvert, the road would act like a dam, blocking the water until it eventually flooded over the top or washed the road away.



A culvert allows that water to pass safely



(illustration from Practical Engineering)

Culverts

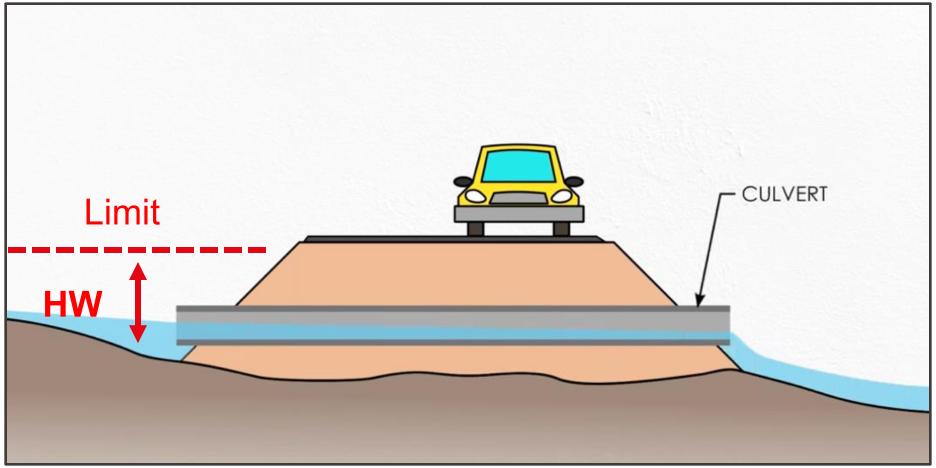
Culverts are hydraulically short closed conduits that convey streamflow through a road embankment or some other type of flow obstruction.

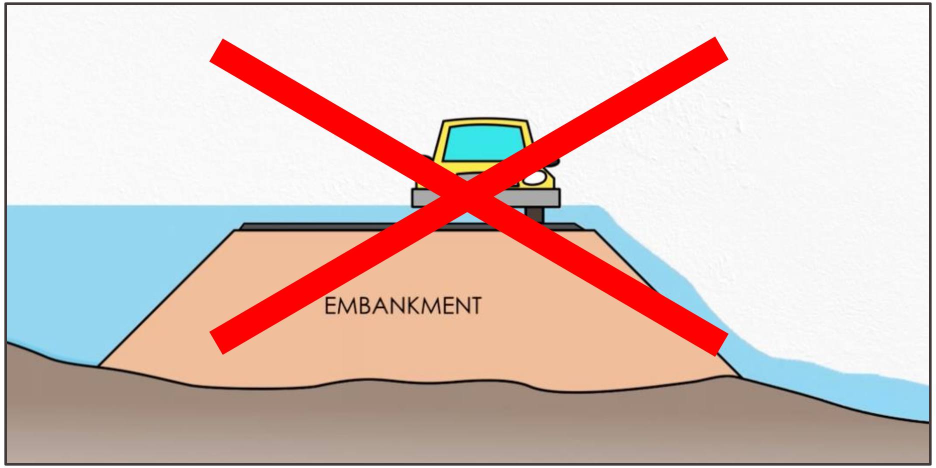
❖ Very complicated hydraulically because they operate in different conditions that vary in time

- Can *flow full* → operates under pressure as in pipe flow
- Can be *partially full* --> operated as an open channel flow
  - OCF can operate in sub- & super-critical, with profile changes including gradually and rapidly varied flow

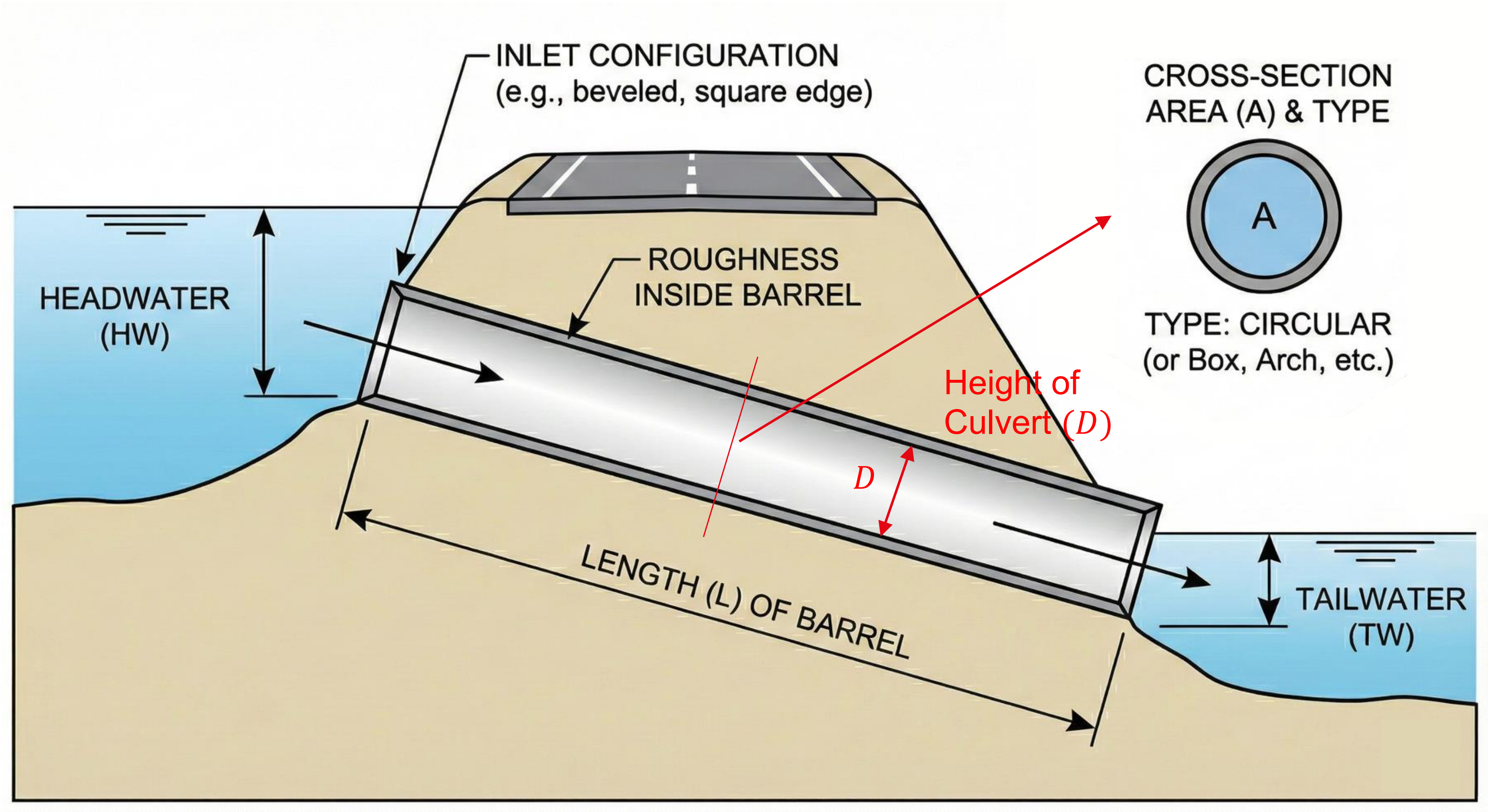
Have ≠ shapes (circular, box, etc.) and material (concrete, aluminum, etc.)

- Culvert design is done by trial analysis → size and geometry are selected and they are analyzed and modified until a satisfactory structure is found
- Culvert design usually is based on the selection of a design discharge Q_d determined from frequency analysis (i.e., based on a specific return time – e.g., “100-year peak Q”)
- The analysis is then to optimize the “best” head-discharge relation $Q = f(H)$ **to limit the *headwater (HW)* resulting from Q_d to a specified value to prevent overtopping whatever is built on top of it (e.g., a street).**





Many factors influence/control the relationship $Q = f(H)$, including:



However, the most important distinguishing characteristic of culvert flow is:

INLET CONTROL

The inlet is the only thing constraining the flow through it, which is shallow and supercritical.

The $H - Q$relation is **only determined by the headwater and the inlet geometry**: area, edge rounding, shape. Tailwater DOES NOT count.

Vs

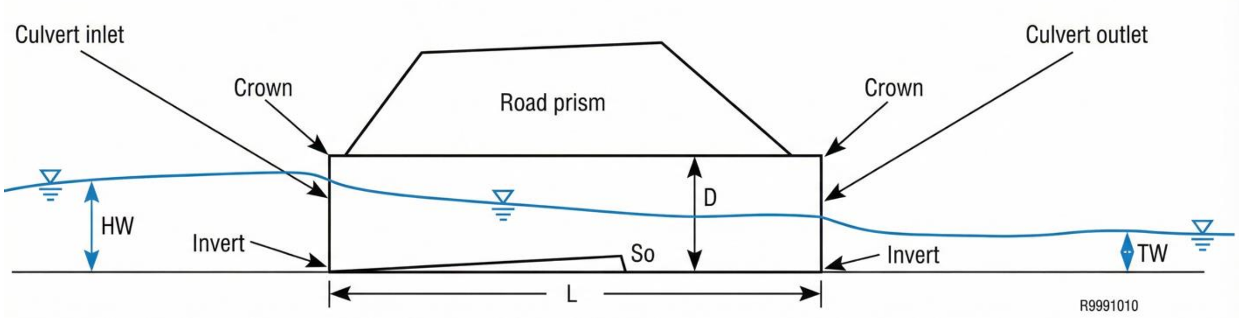
OUTLET CONTROL

The water can flow out faster than it can flow in because limited from downstream.

The $H - Q$relation is affect by the inlet **AND** the barrel roughness, length, slope, shape, area, **AND the Tailwater elevation**

Remember that by control we mean discharge, not water profile, we intend “how Q depends on H”

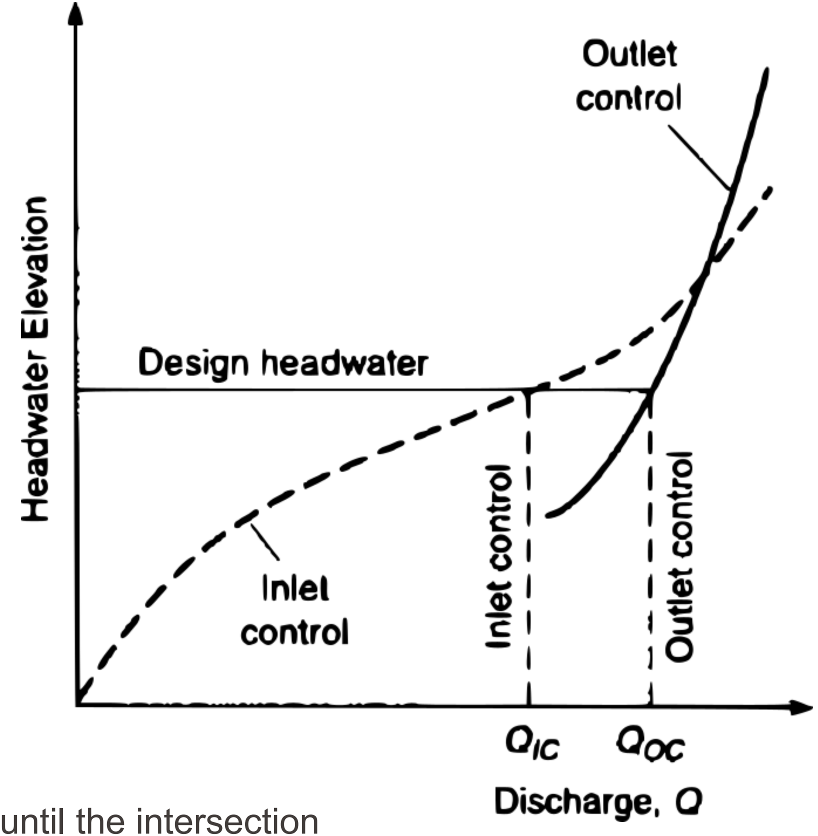
Profile of culvert, inlet control, inlet not submerged, and projecting inlet and outlet



Culvert Performance Curves

Once the design culvert size is determined, its performance must be verified for a range of Q analyzing the complete $H - Q$ relation, to create the ***performance curve*** →

It is generally not possible to establish whether a culvert is IC or OC beforehand; you must analyze both assumptions! → The higher head resulting either from IC or OC is compared with the allowable headwater elevation.

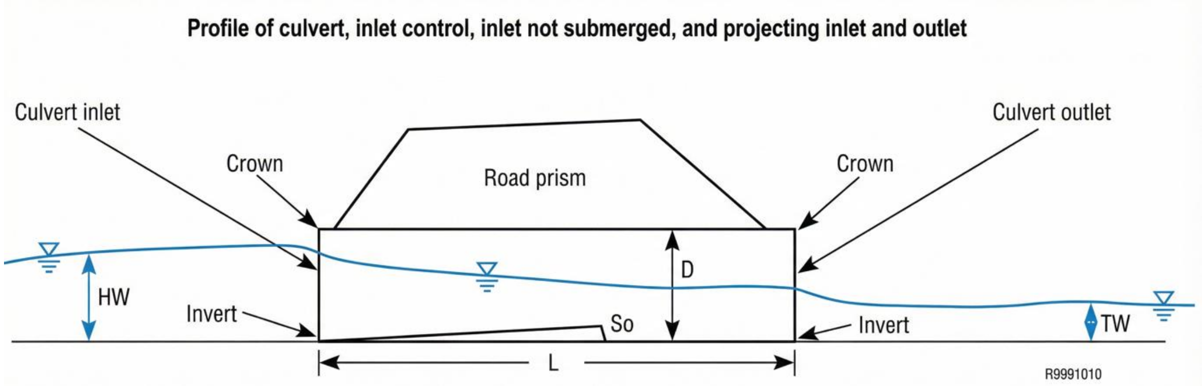


For example, here for a design HW we have two possible Q, each associated to IC or OC. In this case $Q_{IC}(HW) < Q_{OC}(HW)$ so the inlet capacity is less than the barrel capacity and the inlet controls the $H - Q$ relation at the design condition (same as choosing the higher head for a given Q).

As the HW increases, the culvert remains in inlet control until the intersection between the IC and OC curves, beyond which it is assumed to be in outlet control.

The head-discharge relationship $H-Q$ follows well-known hydraulic behavior:

- ***Non-submerged inlet, inlet-controlled*** → operates as WEIR at the inlet → $Q \propto H^{3/2}$
- ***Submerged inlet, inlet controlled*** → operates as ORIFICE at the inlet → $Q \propto H^{1/2}$ (head increases more rapidly with Q than for weir flow – see figure from previous slide)
- ***Pressure-flow*** → $H-Q$ relation determined by the effective head = difference of TOTAL HEAD between HW and TW.



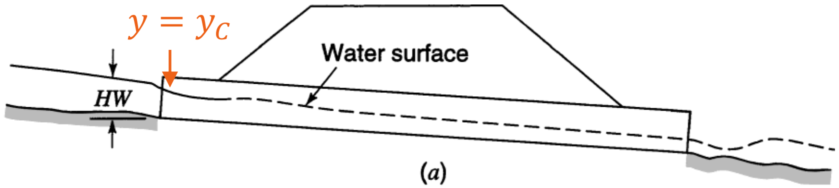
So, most important classification:

- IC vs OC
- Whether the inlet *submerged* or *non-submerged*
  - ❖ *Submergence occurs when:*

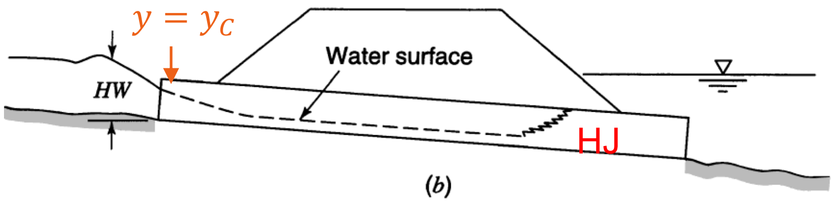
$$\frac{HW}{D} \sim 1.2 - 1.5$$

INLET CONTROL

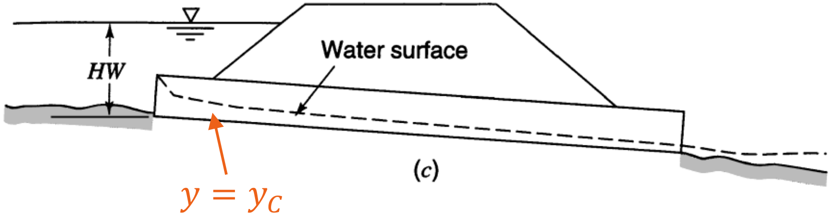
Several types of IC are possible



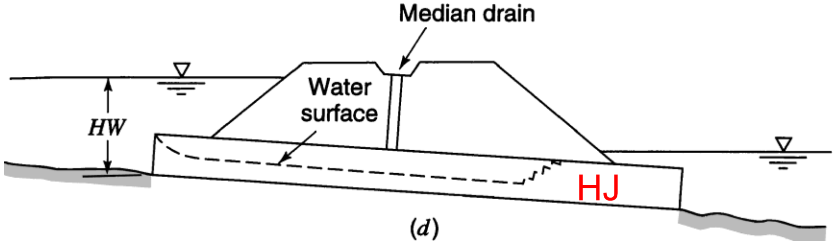
- both inlet and outlet are *non-submerged*
- steep slope → flow passes through critical at the inlet & downstream is $Fr > 1$ → upstream control (coming with S2 profile from critical)



- outlet is submerged and forces a HJ formation inside
- the tail water effect is not strong enough to push the HJ to the inlet so the culvert is not in pressure and the inlet is still controlling.
- $H - Q$ does not change



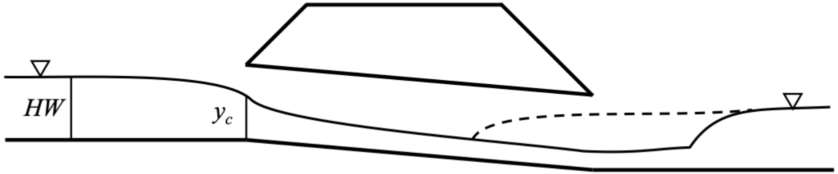
- inlet submerged but outlet non-submerged
- critical depth occurs downstream of inlet and approached normal depths at the downstream end
- culver works in orifice flow
- typical of design conditions



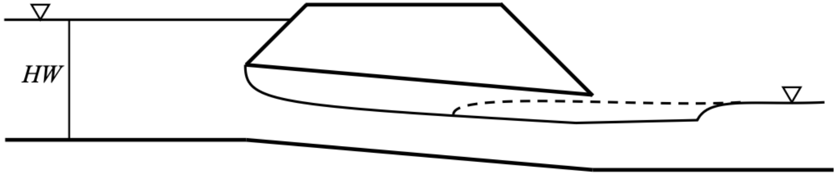
- both inlet and outlet are *submerged*
- a central vent prevents strong oscillations between full flow and partially full flow
- Unstable transition between IC and OC is called slug flow and is more likely if HW/D is between 1.2 and 1.5

INLET CONTROL

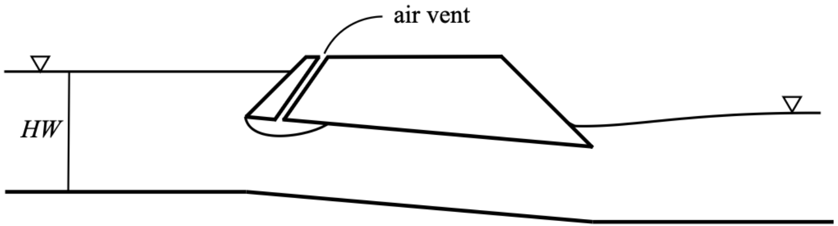
Just another way to visualize all the cases…



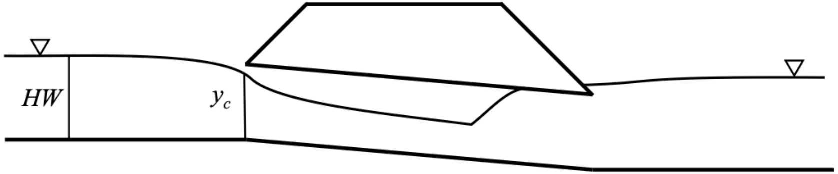
(a) Inlet and outlet unsubmerged



(b) Inlet submerged, outlet unsubmerged



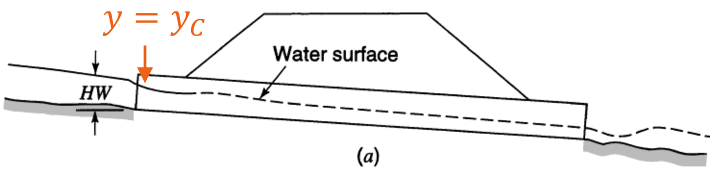
(c) Inlet and outlet submerged

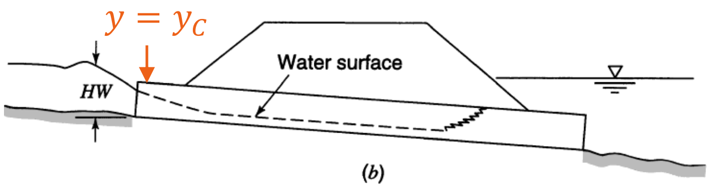


(d) Inlet unsubmerged, outlet submerged

INLET CONTROL → Two head-discharge $H - Q$ relations possible

1) *Inlet non-submerged* → BROAD CRESTED WEIR FLOW





Derived from the energy equation from the HW to the critical depth section, neglecting approaching velocity head:

(K_e: entrance loss coefficient)

$$HW = y_c + (1 + K_e)\frac{Q^2}{2gA_c^2}$$

Critical values

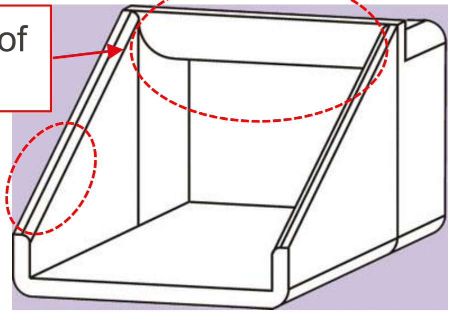
The equation is coupled with $Fr_c^2 = \frac{Q^2}{gA_c^2 y_c} = 1$ to have 2 equations in 2 unknowns (Q, y_c)

The expression can be rewritten as:

$$\boxed{Q = C_d A_c \sqrt{2g(HW - y_c)}}$$

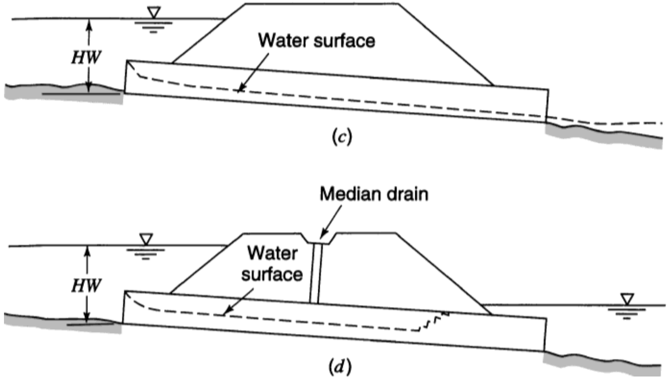
where $c_d = 1/(1 + K_e)^{1/2}$ is the discharge coefficient $f\left(\frac{HW}{D}\right)$, tabulated for different INLET geometry ($C_d = 0.8 - 0.95$)

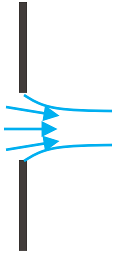
Bevels & rounding of the entrance edge



INLET CONTROL → Two head-discharge $H-Q$ relations possible

1) *Inlet submerged* → ORIFICE FLOW





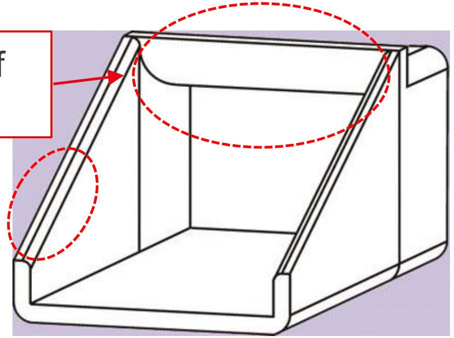
(remember Torricelli?)

Once the inlet is submerged, the governing hydraulic equation is the orifice-flow equation given as

$$Q = C_d A_0 \sqrt{2g(HW)}$$

C_d = coefficient of discharge; A_0 = cross-sectional area of inlet.

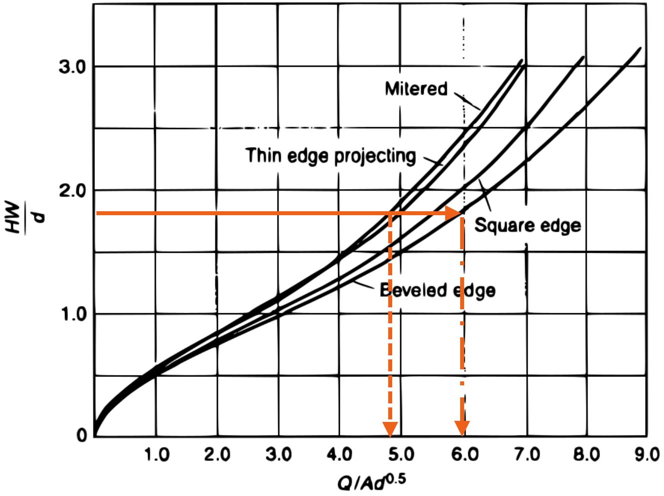
Bevels: rounding of the entrance edge



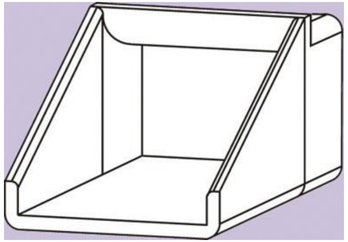
Also in this case, C_d depends strongly on the beveled edges (minimizing dissipation, optimal inclination at 45°) and rounding

→ tabulated values according to rounding with radius r and for bevels of height w as a function of HW/D.

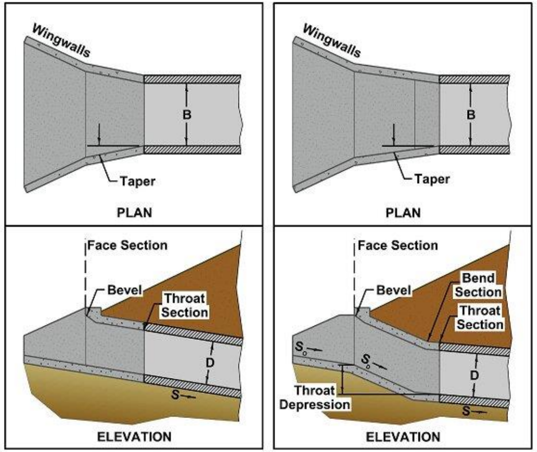
When the culvert is in inlet control, improvements to the discharge capacity can be made by improving the design of the inlet itself to reduce flow contraction, increasing the effective flow area and lowering head losses caused by sharp constrictions.



1) Belving and rounding the entrance



2) Side-tapered and slope-tapered inlet

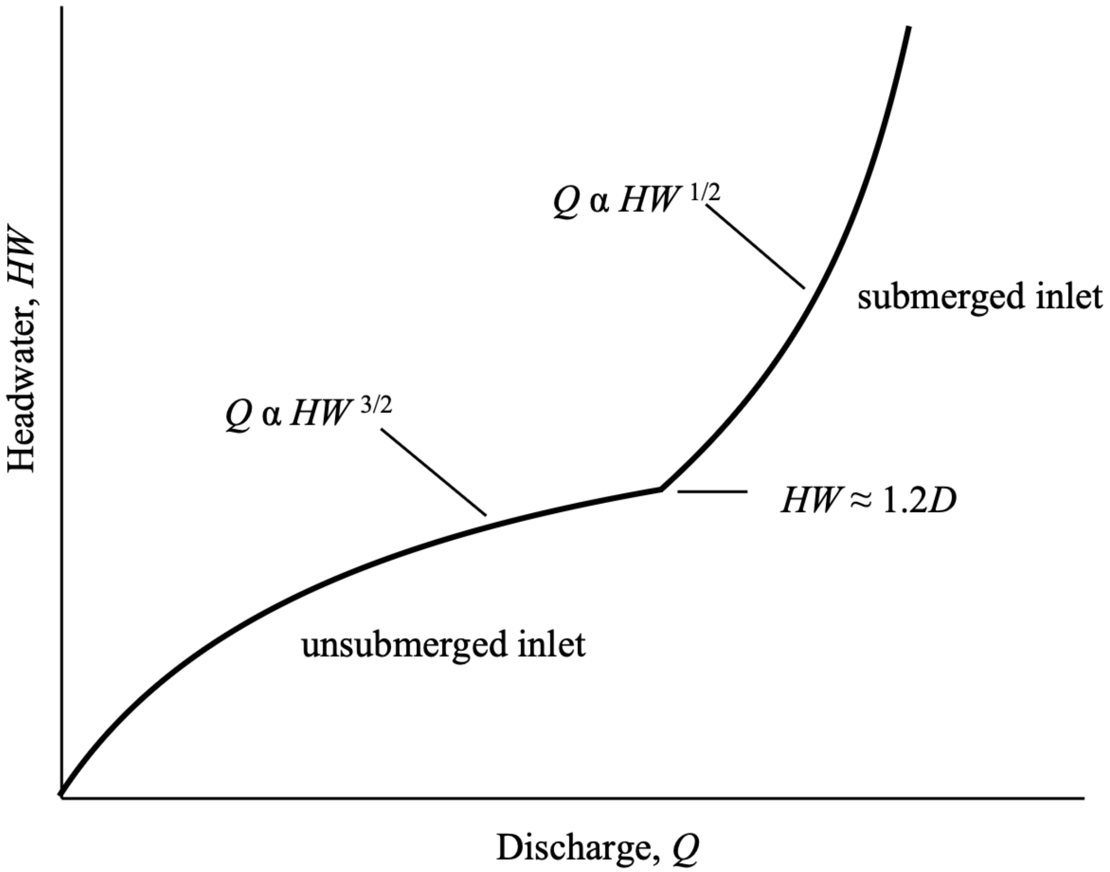


a. Side-tapered inlet.

b. Slope-tapered inlet.

INLET CONTROL

Rating curve for an inlet-controlled box culvert



INLET CONTROL

In order to ensure a smooth transition between the *unsubmerged (open channel / weir flow)* and *submerged (orifice flow)* portions of the inlet control head-discharge equations, these additional experimental equation were developed:

Unsubmerged

- Form 1 (preferred)

$$\frac{HW}{D} = \frac{E_c}{D} + K\left(\frac{Q}{AD^{0.5}}\right)^M - 0.5\ S$$

- Form 2 (easier)

$$\frac{HW}{D} = K\left(\frac{Q}{AD^{0.5}}\right)^M$$

These are applicable for $\frac{Q}{AD^{0.5}} \leq 3.5$

E_c = minimum specific energy (criticality)
A = full cross-sectional area of the barrel
S = culvert barrel slope
K, M = tabulated constants for different types of inlets

These equation were derived by the US National Bureau of Standards hence all the variables are in English units (to reference tabulated constants).

Submerged (orifice-like expression)

$$\frac{HW}{D} = c\left(\frac{Q}{AD^{0.5}}\right)^2 + Y - 0.5\ S$$

Applicable for $\frac{Q}{AD^{0.5}} \geq 4$
c, Y = tabulated constants for different types of inlets

Shape and material	Inlet shape	c	Y	Form (Type 5)	K or K'	M or M'
Circular concrete	Square edge with headwall	0.0398	0.67	1	0.0098	2.0
	Groove end with headwall	0.0292	0.74		0.0018	2.0
	Groove end projecting	0.0317	0.69		0.0045	2.0
Circular CMP*	Headwall	0.0379	0.69	1	0.0078	2.0
	Mitered to slope	0.0463	0.75		0.0210	1.33
	Projecting	0.0553	0.54		0.0340	1.5
Circular	Beveled ring, 45°	0.0300	0.74	1	0.0018	2.5
	Beveled ring, 33.7°	0.0243	0.83		0.0018	2.5
Rectangular box, concrete	30°–75° wingwall flares	0.0347	0.81	1	0.026	1.0
	30° and 75° wingwall flares	0.0400	0.80		0.061	0.75
	0° wingwall flares	0.0423	0.82		0.061	0.75
Rectangular box, concrete	45° wingwall flare, $w/D = 0.043$	0.0309	0.80	2	0.510	0.667
	18° to 33.7° wingwall flare, $w/D = 0.083$	0.0249	0.83		0.486	0.667
	90° headwall, 19-mm chamfers	0.0375	0.79		0.515	0.667
	90° headwall, 45° bevels	0.0314	0.82		0.495	0.667
	90° headwall, 33.7° bevels	0.0252	0.865		0.486	0.667
	19-mm chamfers, 45° skewed headwall	0.04505	0.73		0.545	0.667
	19-mm chamfers, 30° skewed headwall	0.0425	0.705		0.533	0.667
	19-mm chamfers, 15° skewed headwall	0.0402	0.68		0.522	0.667
	45° bevels, 10°–45° skewed headwall	0.0327	0.75		0.498	0.667
	19-mm chamfers, 45° wingwall flare, nonoffset	0.0339	0.803		0.497	0.667
	19-mm chamfers, 18.4° wingwall flare, nonoffset	0.0361	0.806		0.493	0.667
	19-mm chamfers, 18.4° wingwall flare, nonoffset, 30° skew	0.0386	0.71		0.495	0.667
	Top bevels, 45° wingwall flare, offset	0.0302	0.835		0.497	0.667
	Top bevels, 33.7° wingwall flare, offset	0.0252	0.881		0.495	0.667
	Top bevels, 18.4° wingwall flare, offset	0.0227	0.887		0.493	0.667
Box CM	90° headwall	0.0379	0.69	1	0.0083	2.0
	Thick wall projecting	0.0419	0.64		0.0145	1.75
	Thin wall projecting	0.0496	0.57		0.0340	1.5

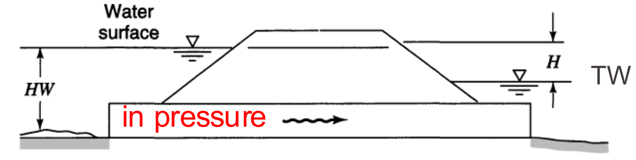
Shape and material	Inlet shape	c	Y	Form (Type 5)	K or K'	M or M'
Ellipse concrete	Horizontal ellipse, square edge with headwall	0.0398	0.67	1	0.0100	2.0
	Horizontal ellipse, groove end with headwall	0.0292	0.74		0.0018	2.5
	Horizontal ellipse, groove end projecting	0.0317	0.69		0.0045	2.0
	Vertical ellipse, square edge with headwall	0.0398	0.67		0.010	2.0
	Vertical ellipse, groove end with headwall	0.0292	0.74		0.0018	2.5
	Vertical ellipse, groove end projecting	0.0317	0.69		0.0095	2.0
Arch CM†	46-cm corner radius, 90° headwall	0.0379	0.69	1	0.0083	2.0
	46-cm corner radius, mitered to slope	0.0463	0.75		0.0300	1.0
	46-cm corner radius, projecting	0.0496	0.57		0.0340	1.5
	46-cm corner radius, projecting	0.0496	0.57		0.0300	1.5
	46-cm corner radius, no bevels	0.0368	0.68		0.0088	2.0
	46-cm corner radius, 33.7° bevels	0.0269	0.77		0.0030	2.0
	79-cm corner radius, projecting	0.0496	0.57		0.0300	1.5
	79-cm corner radius, no bevels	0.0368	0.68		0.0088	2.0
	79-cm corner radius, 33.7° bevels	0.0269	0.77		0.0030	2.0
	90° headwall	0.0379	0.69		0.0083	2.0
	Mitered to slope	0.0473	0.75		0.0300	1.0
	Thin-wall projecting	0.0496	0.57		0.0340	1.5
Circular	Smooth-tapered inlet throat	0.0196	0.90	2	0.534	0.555
	Rough-tapered inlet throat	0.0210	0.90		0.519	0.64
Elliptical inlet face	Tapered inlet, beveled edges	0.0368	0.83	2	0.536	0.622
	Tapered inlet, square edges	0.0478	0.80		0.5035	0.719
	Tapered inlet, thin edge projecting	0.0598	0.75		0.547	0.80
Rectangular concrete	Tapered inlet throat	0.0179	0.97	2	0.475	0.667
Rectangular concrete	Side tapered, less favorable edge	0.0446	0.85	2	0.56	0.667
	Side tapered, more favorable edge	0.0378	0.87		0.56	0.667
Rectangular concrete	Side tapered, less favorable edge	0.0446	0.65	2	0.50	0.667
	Side tapered, more favorable edge	0.0378	0.71		0.50	0.667

Source: U.S. Federal Highway Administration (2012).

Notes: *CMP = corrugated metal pipe; †CM = corrugated metal.

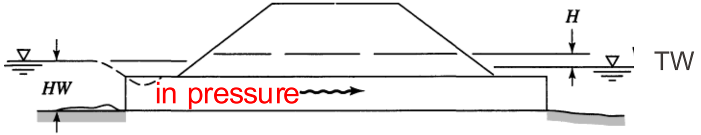
OUTLET CONTROL

Several types of OC are possible



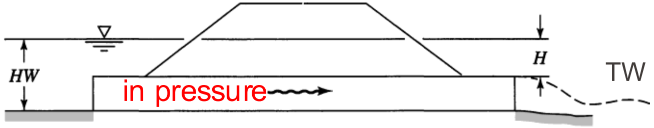
(a)

- Full pipe → pressure flow occurs throughout the entire barrel



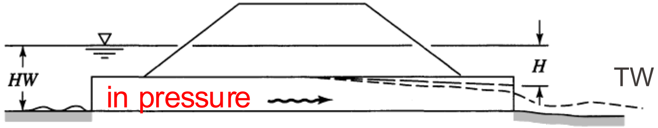
(b)

- the outlet is submerged but the inlet is unsubmerged for low values of headwater because of the flow contraction at the inlet



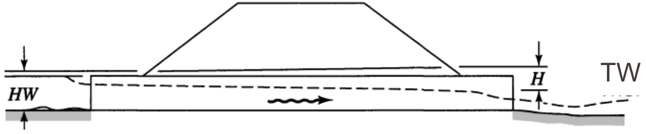
(c)

- the outlet is unsubmerged but the culvert still flows full pipe because HW is very high



(d)

- not only the outlet unsubmerged, but the barrel flows also partly full (open channel flow) near the outlet and passes through critical depth there



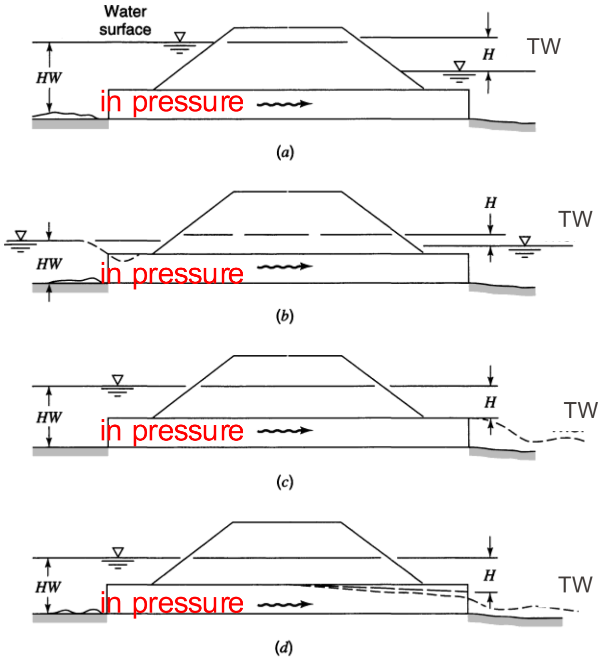
(e)

- both the inlet and outlet are unsubmerged and we have open channel flow that is subcritical on a mild slope (hence controlled downstream)

OUTLET CONTROL

Head-discharge $H-Q$ relations possible

1) First type: all treated as full flow (pressurized conduit) with only some adjustment for the last type (d).



The discharge expression can be developed by accounting for all the energy losses up to the control point → Governed by the energy equation written from the head water (HW) to the tail water (TW):

$$HW = TW - S_0 L + \left(1 + K_e + f\frac{L}{4R}\right)\frac{Q^2}{2gA^2}$$

TW = tailwater depth relative to the outlet invert; S_0 = culvert slope; L = culvert length; K_e = entrance loss coefficient; f = Darcy-Weisbach friction factor; R = full-flow hydraulic radius; A = culvert cross-sectional area; and Q = culvert discharge.

Rearranging:

$$Q = A\sqrt{\frac{2g(HW - TW + S_0 L)}{1 + K_e + f\frac{L}{4R}}}$$

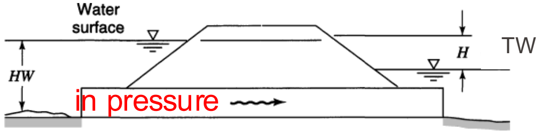
OUTLET CONTROL

Head-discharge $H - Q$ relations possible

1) First type: all treated as full flow (pressurized conduit) with only some adjustment for the last type (d).

$$Q = A\sqrt{\frac{2g(HW - TW + S_0 L)}{1 + K_e + f\frac{L}{4R}}}$$

$(HW - TW + S_0 L)$ = Effective head (H in figure)



K_e is the entrance loss coefficient for outlet control also tabulated for outlet type

Head-loss term can be rewritten in terms of Manning's equation

$$\frac{fL}{4R} = \frac{2gn^2L}{R^{4/3}}$$

and n values are tabulated for different culvert conditions (debris and aging effects) and often used to get the friction factor

Culvert Entrance Loss Coefficients

Recommended Manning's n in Culverts

Type of conduit	Wall and joint description	n
Concrete pipe	Good joints, smooth walls	0.011–0.013
	Good joints, rough walls	0.014–0.016
	Poor joints, rough walls	0.016–0.017
	Badly spalled	0.015–0.020
Concrete box	Good joints, smooth, finished walls	0.012–0.015
	Poor joints, rough, unfinished walls	0.014–0.018
Spiral rib metal pipe	19-mm × 19-mm recesses at 30-cm spacing, good joints	0.012–0.013
Corrugated metal pipe, pipe arch, and box	68-mm × 13-mm annular corrugations	0.022–0.027
	68-mm × 13-mm helical corrugations	0.011–0.023
	150-mm × 25-mm helical corrugations	0.022–0.025
	125-mm × 25-mm corrugations	0.025–0.026
	75-mm × 25-mm corrugations	0.027–0.028
	150-mm × 50-mm structural plate	0.033–0.035
	230-mm × 64-mm structural plate	0.033–0.037
Polyethylene	Corrugated	0.018–0.025
	Smooth	0.009–0.015
PVC	Smooth	0.009–0.011

Source: U.S. Federal Highway Administration (2005a; 2012).

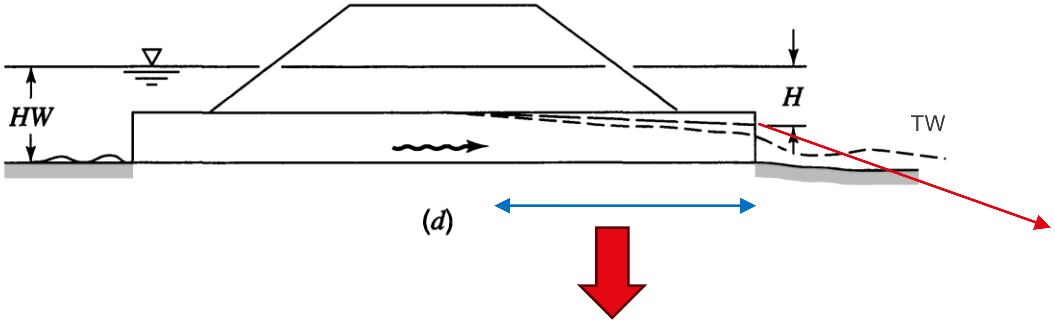
Culvert type and entrance conditions	k_e
Pipe, concrete:	
Projecting from fill, socket end (groove end)	0.2
Projecting from fill, square-cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove end)	0.2
Square edge	0.5
Rounded (radius = $D/12$)	0.2
Mitered to conform to fill slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Pipe, or pipe arch, corrugated metal:	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls, square edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Box, reinforced concrete:	
Headwall parallel to embankment (no wingwalls)	
Square edged on 3 edges	0.5
Rounded on 3 edges	0.2
Wingwalls at 30° to 75° to barrel	
Square edged at crown	0.4
Crown edge rounded	0.2
Wingwalls at 10° to 25° to barrel	
Square edged at crown	0.5
Wingwalls parallel (extension of sides)	
Square edged at crown	0.7
Side or slope-tapered inlet	0.2

Source: U.S. Federal Highway Administration (2012).

OUTLET CONTROL

Head-discharge $H - Q$ relations possible

1) Case (d) is particular and require special attention.



$$\frac{y_c + D}{2}$$

It requires the computation of the subcritical flow profile from the outlet to the top of of the culvert.

Alternative procedure:

1) assume a full flow from INLET to OUTLET with $TW = \frac{y_c + D}{2}$
2) compute the head water HW using the full flow equation from before

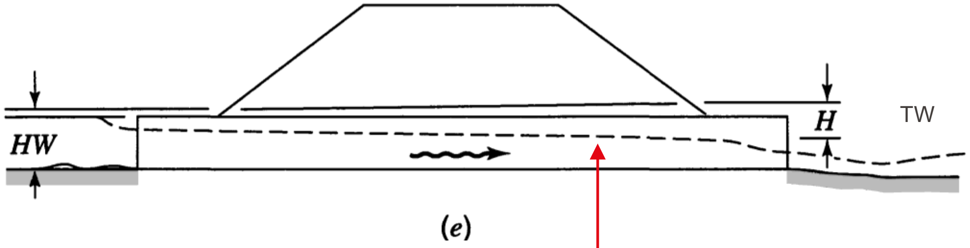
$$HW = \frac{y_c + D}{2} - S_0 L + \left(1 + K_e + f\frac{L}{4R}\right)\frac{Q^2}{2gA^2}$$

3) compare this estimate with the given B.C. $HW_{B.C.}$ → if HW $< HW_{B.C.}$ it means that $HW_{B.C.}$ does not depend on the tailwater $TW = \frac{y_c + D}{2}$ and the culvert **is NOT outlet control**

OUTLET CONTROL

Head-discharge $H-Q$ relations possible

2) Second type is an open channel flow



It requires the computation of a gradually varied flow profile from the outlet proceeding upstream to the culvert inlet. This will be either an M2 or an M1 profile. At the inlet, the velocity head and entrance losses k_e from the table in slide 45 are added to the inlet flow depth to obtain the upstream headwater, HW